Inheritance, Land, and Capital Mobility linked to Labour Mobility

Damien Gaumont *  Alice Mesnard **

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* Université Paris 2 and ERMES, CNRS, UPRES-A 7017, 92, rue d’Assas, 75270 Paris Cedex 06, France. damien.gaumont@free.fr

** ARQADE 21 Allée de Brienne, 31 000 Toulouse, France and Université Toulouse 1

Abstract

This paper presents a two-country migration model, following Galor (1986), in which the world population consists of two types individuals. Individuals with a high (low) degree of altruism give to their children a high (low) level of bequest. Production uses three inputs: immobile land, mobile labour, and capital. Capital mobility is linked to labour mobility since individuals move with their inheritance. The model shows that countries are homothetic in the post-migration equilibrium with equal factor prices and equal densities of population. Migration flows are bilateral and the number of each type of migrants is uniquely determined. In some cases, migration leads to a Pareto improvement in both countries.

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1All correspondance to Alice Mesnard: alice.mesnard@univ-tlse1.fr
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1 Introduction

This paper explores the theoretical link between wealth distribution, migration and density of population. Our main interest is to provide a better understanding of the determination of migration flows across countries. It is shown that differences in the distribution of wealth due to differences in altruism determines migration flows and leads both economies to be inhabited after migration is permitted, as is usually observed. Taking into account the role of land allows us to determine the number of each type of migrant and, in some cases, we can show that a Pareto improvement results from migration.

One of the major motivations to study the case of capital mobility linked to labour mobility comes from casual observation. In the context of international migration of Tunisian workers, Mesnard (1999) provides evidence that savings are brought back to Tunisia by migrants. Moreover in some countries such as Israel, holding capital outside the country is forbidden, (see Darvish-Lecker & Kahana (1992)). These observations motivate us to adopt the assumption that individuals migrate with their wealth. Consequently migration modifies the wealth distribution in each country. In order to generate wealth distributions, Loury (1981) and Galor and Zeira (1996) assume various degrees of altruism among individuals. Following them, we examine a dynamic general equilibrium model with overlapping generations where differences in individuals’ degrees of paternalistic altruism permit us to study the role of wealth distribution on migration.

Overlapping generations models have been often used to explain international migration. Following the seminal model of Galor (1986), a wide class of models studies the aggregate implications of life-cycle saving by heterogeneous agents on migration. Different time preferences across countries lead to different intensities of capital accumulation and a trade-off between the wage differential and the interest rate differential generates migration flows\(^2\). This class of models offers a framework in general equilibrium to compare the welfare implications of labour versus capital mobility, (see Galor (1992)). This class of models has been extended in two directions. Firstly, still considering two factors of production, Kondo (1989), and Galor and Stark (1991) show that differences across countries in technologies of production (instead of in time preferences) also generate price differentials between the economies in autarky inciting workers to migrate\(^3\). Vidal (1998) shows, in a small open economy, that emigration may induce growth by inciting human capital accumulation in the source country and this can lead the sending country out of a poverty trap. Kochhar (1992) introduces endogenous labour supply to analyse the responsiveness of domestic labour supply and wage rates to immigration. Secondly, recent extensions of Galor (1986) consider three factors of production, capital, labour and land, (see Karayalcin (1994), and Cretetze, Michel and Vidal (1996)). In that context Cretetze, Michel and Vidal (1998) compare capital mobility and labour mobility. Results are modified as migration ceases when population densities and interest rates are equal across countries. Moreover welfare analysis is also modified by a new determinant of individuals’ indirect utilities: the density of population.

\(^2\)Labour migrates from high (low) to the low (high) time preference country if both countries under (over) invest relative to the Golden Rule. Unilateral migration worsens (improves) the welfare of non-migrants in the immigration (emigration) country. Bilateral migration improves (worsens) the welfare of non-migrants that are characterized by identical (different) preferences to those of the migrants from their country.

\(^3\)Migrants move from the technological superior (inferior) country if and only if the autarkic steady-state equilibrium is characterized by over (under) investment relative to the Golden Rule.
Another direction of the literature analyses the aggregate implications of intergenerational transfers through bequests. Surprisingly only few papers study migration in this framework. In *partial equilibrium* Tcha (1995) studies rural-urban migration with altruistic individuals. Differences in *dynastic* utility among young and old family members are the source of higher mobility by the young, and this generates conflicts between generations about migration decisions. To avoid these problems, Gaumont and Mesnard (2000) built an overlapping generation *general equilibrium* model with *paternalistic* altruism. Parent’s utility takes into account the bequest to the child instead of the whole utility of the child.

The present paper is at the intersection of these two strands of overlapping generation models. It investigates the effects of paternalistic altruism on international labour migration in a dynamic general equilibrium model with overlapping generations and three factors of production. We present a symmetric case to Crettez, Michel and Vidal (1996) where migrants move with their inheritance. In overlapping generation models, bequests motivated by altruism better capture the empirical fact of *inter vivos* gifts between relatives than the concept of “inheritance” as usually labeled afterwards, (see Barro (1974), Buitr (1979), Carmichael (1982) and Weil (1987)). Indeed individuals receive bequests when young from their old parents and not when old after their parents’ death. For this reason, we mean by “inheritance” *inter vivos* gifts motivated by altruism. *Inter vivos* gifts are common occurrences in both developed and developing countries and in some cases constitute the initial migrant’s wealth prior migration4.

In our model, we consider that transfers of capital and land from parent to child take place before migration. Individuals are heterogeneous with respect to their degree of paternalistic altruism towards children. During childhood an individual saves her entire inheritance and, when old, she works, consumes and chooses the level of her child’s inheritance. Her income consists of wage income and the return on the inheritance received from her parents. Firms act competitively. When borders are open, at the autarkic steady-state equilibrium, incentives for migration take place. It is shown that bilateral migration leads both economies to be inhabited in post-migration equilibrium. Indeed, post-migration equilibrium is characterized by equal proportions of individuals of each type and equal factor prices across countries. Moreover, introducing land enables us to resolve an undesirable feature of Gaumont and Mesnard (2000). The number of individuals having migrated to each other economy is now fully determined since bilateral migration flows leads to the equalization of population densities across countries5.

Our welfare implications are generally ambiguous for some individuals since two effects come into play. One is working through the density of population, the other one through the interest rate. Nevertheless it is possible to characterize the impact of labour mobility on individual welfare in two extreme cases. The first deals with the pattern of international labour

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4 About intrahousehold transfers, see Gale and Sholtz (1994) for USA, Knowles and Anker (1981) for Kenya, Lucas and Stark (1985) for Botswana, Butz and Stan (1982) for Malaysia, Weinstein and al (1994), Lee, Parish and Willis (1994) for Taiwan, and Secondi (1997) for China. Two competing hypotheses have been previously advanced to justify *inter vivos* transfers: altruism or exchange of services. Empirical studies have produced conflicting results while testing these two hypotheses. The ambiguity of these results is in part due to a relative scarcity of quality data, Cox, Exer and Jimenez (1998). When migrants intend to stay in the host country, Poirem (1997) provides evidence that remittances cease after a while. Observations of altruistic *inter vivos* gifts are consistent with the traditional overlapping generation formalization of intergenerational transfers through inheritance.

5 In contrast to Galor’s (1986), our results with different degrees of paternalistic altruism do not depend on the Golden Rule.
mation when both countries exhibit the same density of population before migration is permitted. The second examines the steady-state welfare implications of labour mobility when both countries exhibit the same steady-state interest rate in autarky. The steady-state welfare implications of world integration for each type of individual in each country depend on both effects. In some cases a comparative welfare analysis between autarkic steady-state equilibrium and post-migration steady-state equilibrium shows that world integration may be Pareto-improving.

This paper is organized as follows. Section 2 presents the model in autarky. Section 3 analyses international labour migration. Section 4 provides a welfare analysis. Section 5 discusses some assumptions of the model and Section 6 concludes.

2 Autarky

Consider a two-country world with an infinite horizon, discrete time and no uncertainty. The technology is the same in each country, \( i = 1, 2 \). In period \( t \), three factors of production, labour quantity \( L_i^t \), capital quantity \( K_i^t \), and land quantity \( X_i^t \), produce a single good. The prices of labour, capital and land are respectively \( w_i^t \), \( r_i^t \) and \( q_i^t \). Output is the numeraire and is allocated to current consumption \( C_i^t \) and total investment (in both capital and land) in the next period, \( I_{i+1}^t \). Thus, a price system is a positive vector \( p_i^t = (w_i^t, R_i^t, q_i^t) \), where \( R_i^t = 1 + r_i^t \) represents the interest factor. For each country, the endowment of labour in each period is exogenous, whereas capital is the output produced but not consumed in the preceding period. Capital fully depreciates after one period. Land is a fixed factor of production.

During each period \( T \) individuals are born in country \( i \). Each adult has only one child. The child inherits her parent’s degree of altruism. There is zero-population growth. The number of individuals of type \( j \) in country \( i \) is \( N^{i,j} \), where \( j = 1, 2 \) is the degree of altruism.

The decision process of an individual who lives for two periods, childhood and adulthood, is the following. After receiving her inheritance in period \( t-1 \), the child saves it completely. During childhood, she does not consume and does not work. Individuals born at \( t-1 \) derive utility from consumption \( C_i^{t,j} \) and bequest \( A_i^{t,j} \) in period \( t \). Hence, individuals save their entire inherited wealth composed of capital and land \( A_i^{t,j} = K_i^{t,j-1} + X_i^{t,j-1} \). They can invest their savings in land or capital so that the total return on land equalizes that on capital. The asset market equilibrium equation is

\[
R_i^t = \frac{q_i^t}{q_i^{t-1}} + \pi_i^t, \quad i = 1, 2,
\]

where \( \pi_i^t \) is the return of the land.

An individual supplies one unit of labour at the competitive wage \( w_i^t \) and receives inheritance return \( R_i^t A_i^{t,j} \). She allocates her income between her consumption \( C_i^{t,j} \) and her bequest \( A_i^{t,j} \) to maximize her life-time utility function. The budget constraint of the individual \( j \) in country \( i \) is \( \Omega_i^{t,j} = C_i^{t,j} + A_i^{t,j} \), where the life-cycle income, \( \Omega_i^{t,j} \), is equal to \( w_i^t + R_i^t A_i^{t,j-1} \).

\[\text{In the remainder of the paper population is not indexed by time except when necessary to describe the post-migration equilibrium (section 3)}\]
An individual is characterized by her life-time utility. Preferences are homothetic, homogeneous of degree 1 and individual \( j \) solves

\[
\begin{aligned}
\max_{C_t^{i,j}, A_t^{i,j}} & \quad U^j(C_t^{i,j}, A_t^{i,j}) \\
\text{s.t.} & \quad \Omega_t^{i,j} = C_t^{i,j} + A_t^{i,j} \\
& \quad C_t^{i,j} \geq 0, \quad A_t^{i,j} \geq 0.
\end{aligned}
\]

Optimal choices are given by

\[
A_t^{i,j} = \alpha_j \Omega_t^{i,j},
\]

\[
C_t^{i,j} = (1 - \alpha_j) \Omega_t^{i,j},
\]

where (3) represents the wealth transmission through time which is an increasing function of the degree of altruism. The weight \( \alpha_j \) is a function of the degree of altruism of an individual of type \( j \). Since the utility function is homogeneous of degree 1, the indirect utility \( \bar{U}^j \) is

\[
\bar{U}^j(\alpha_j \Omega_t^i, (1 - \alpha_j) \Omega_t^i) = \Omega_t^j \bar{U}^j(\alpha_j, 1 - \alpha_j),
\]

where \( \alpha_j \) is determined as \( \bar{U}^j(\alpha_j, 1 - \alpha_j) \). For simplicity we note \( \alpha_1 = a \) and \( \alpha_2 = b \) with \( a > b \): “type-a" individuals are more altruistic than “type-b" individuals. In period \( t \) consumption (4) is a decreasing function of the degree of altruism. Saving in period \( t \) is \( A_t^{i,j} = \Omega_t^{i,j} - C_t^{i,j} \).

Production in each country occurs within a period according to a Cobb-Douglas production function independent of time and invariant across countries:

\[
F(K_t^i, L_t^i, X_t^i) = (K_t^i)^{\alpha} (L_t^i)^{\beta} (X_t^i)^{1-\alpha-\beta}, \quad \text{where } \alpha > 0, \beta > 0 \text{ and } \alpha + \beta < 1.
\]

Firms act competitively. At prices \( p_t^i = (w_t^i, R_t^i, q_t^i) \) the demand for factors of production in country \( i \) are chosen to maximize profits, which yields successively

\[
w_t^i = F_L(K_t^i, L_t^i, X_t^i) = \beta(K_t^i)^{\alpha}(L_t^i)^{\beta-1}(X_t^i)^{1-\alpha-\beta} = \beta \frac{F}{L_t^i},
\]

\[
R_t^i = F_K(K_t^i, L_t^i, X_t^i) = \alpha(K_t^i)^{\alpha-1}(L_t^i)^{\beta}(X_t^i)^{1-\alpha-\beta} = \alpha \frac{F}{K_t^i},
\]

\[
\pi_t^i = F_X(K_t^i, L_t^i, X_t^i) = (1 - \alpha - \beta)(K_t^i)^{\alpha}(L_t^i)^{\beta}(X_t^i)^{-\alpha-\beta} = (1 - \alpha - \beta) \frac{F}{X_t^i}.
\]

In equilibrium, the demand for land, \( X_t^i \), is equal to land endowment \( X_t \). The demand for labour is equal to supply which amounts to the total number of individuals born at time \( t \)

\[
\bar{L}^i = L_t^i \quad \text{with} \quad \bar{L} = N^{i,1} + N^{i,2};
\]

\[
X_t^i = X_t.
\]

Aggregate saving of period \( t \) is used to buy capital and land of period \( t+1 \). The dynamics of capital accumulation is thus given by

\[
q_t^i X_t + K_t^i = N_i^{i,1} A_t^{i,1} + N_i^{i,2} A_t^{i,2}.
\]

Let us define \( \tilde{k}_t^i = K_t^i/X_t^i \) the capital-density and \( d_t^i = L_t^i/X_t^i \) the population-density. Consequently the capital-labour ratio is \( k_t^i = \tilde{k}_t^i/d_t^i \).
2.1 The steady-state equilibrium in autarky

We now turn to characterize the steady-state equilibrium in autarky.

**Proposition 1** There exists a unique steady state equilibrium. The steady-state interest rate is a decreasing function of the proportion of the more altruistic type individuals.

**Proof:** The steady state is characterized by the following equations

\[ w^i = \beta (\tilde{k}^i)^{\alpha (d^i)^{\beta-1}}, \]  \hspace{1cm} (12)

\[ R^i = \alpha (\tilde{k}^i)^{\alpha - 1 (d^i)^{\beta}}, \]  \hspace{1cm} (13)

\[ \pi^i = (1 - \alpha - \beta)(\tilde{k}^i)^{\alpha (d^i)^{\beta}}, \]  \hspace{1cm} (14)

\[ L^i = N^{i,1} + N^{i,2}, \]  \hspace{1cm} (15)

\[ \pi^i = q^i (R^i - 1), \quad i = 1, 2, \]  \hspace{1cm} (16)

\[ \Omega^{i,1} = \frac{w^i}{1 - aR^i}, \quad \Omega^{i,2} = \frac{w^i}{1 - bR^i}, \]  \hspace{1cm} (17)

\[ q^i X^i + K^i = N^{i,1}A^{i,1} + N^{i,2}A^{i,2}. \]  \hspace{1cm} (18)

Since \( a > b, \Omega^{i,j} > 0 \) and from (17) we obtain \( 1 < R^i < 1/a \). From (16), (17) and (18) the steady-state interest rate fulfills necessarily

\[ \frac{\pi^i X^i}{R^i - 1} + K^i = w^i (\frac{a N^{i,1}}{1 - aR^i} + \frac{b N^{i,2}}{1 - bR^i}). \]

Using (6), (7) and (8) we can write

\[ \frac{\pi(1 - \alpha - \beta)F}{R^i - 1} + \frac{\alpha F}{R^i} = \frac{\beta F}{L} (\frac{a N^{i,1}}{1 - aR^i} + \frac{b N^{i,2}}{1 - bR^i}). \]  \hspace{1cm} (19)

We simplify by \( F \) and multiply by \( R/\alpha \) to obtain

\[ 1 + \frac{\pi(1 - \alpha - \beta)R^i}{\alpha(R^i - 1)} = \frac{\beta a \gamma^i R^i}{\alpha(1 - aR^i)} + \frac{\beta b(1 - \gamma^i)R^i}{\alpha(1 - bR^i)}. \]

Let us note \( \gamma^i = N^{i,1}/L \) the proportion of the more altruistic type individuals in country \( i \) and define

\[ P(R^i, \gamma^i) \equiv 1 + \frac{\pi(1 - \alpha - \beta)R^i}{\alpha(R^i - 1)} - \frac{\beta a \gamma^i R^i}{\alpha(1 - aR^i)} - \frac{\beta b(1 - \gamma^i)R^i}{\alpha(1 - bR^i)}. \]

\[ P(R^i, \gamma^i) = 0 \iff Z(R^i, \gamma^i) = 0 \]
where
\[
Z(R^i, \gamma^i) \equiv \alpha(1 - aR^i)(1 - bR^i)(R^i - 1) + \pi(1 - \alpha - \beta)R^i(1 - aR^i)(1 - bR^i)
\]
\[
- \beta R^i(R^i - 1) \left[ a\gamma^i(1 - bR^i) - b(1 - \gamma^i)(1 - aR^i) \right] = 0.
\]
(20)
\[
Z(1, \gamma^i) = \pi(1 - \alpha - \beta)(1 - a)(1 - b) > 0,
\]
\[
Z(1/a, \gamma^i) = -\left( \frac{1}{a} - 1 \right)(\frac{b}{a} - 1)\gamma^i \beta < 0.
\]
Since \( Z(1, \gamma^i) > 0 \) and \( Z(1/a, \gamma^i) < 0 \), there exists at least one steady-state interest rate. We now demonstrate the unicity of the steady-state. In order to do so, we show that \( P(R^i, \gamma^i) \) is monotone on the interval of definition of \( R^i \).
\[
\frac{\partial P(R^i, \gamma^i)}{\partial R^i} = -\frac{\pi(1 - \alpha - \beta)}{\alpha(R^i - 1)^2} - \frac{\beta a\gamma^i}{\alpha(1 - aR^i)^2} = -\frac{\beta b(1 - \gamma^i)}{\alpha(1 - bR^i)^2} < 0.
\]
Since \( \forall R^i \in [1, 1/a] \) \( \partial P/\partial R^i < 0 \), the steady state interest rate is unique. \( \square \)

**Lemma 1** The interest factor is a decreasing function of the proportion of the more altruistic type individuals and an increasing function of the wage.

Proof: It is easy to show that \( \partial P(R^i, \gamma^i)/\partial \gamma^i = (b - a)(R^i - 1)\beta R^i < 0 \) since \( a > b \). Differentiating \( P(R^i, \gamma^i) = 0 \) we have:
\[
\frac{\partial P(R^i, \gamma^i)}{\partial \gamma^i} d\gamma^i + \frac{\partial P(R^i, \gamma^i)}{\partial R^i} dR^i = 0 \Rightarrow \frac{dR^i}{d\gamma^i} = \frac{\partial P(R^i, \gamma^i)}{\partial R^i} \times \frac{\partial R^i}{\partial P(R^i, \gamma^i)}.
\]
Consequently \( dR^i/d\gamma^i < 0 \) which implies \( R^1 < R^2 \). From (13) \( dR^i/d\gamma^i = a(d^i)^\beta \frac{\partial (\tilde{k})}{\partial (\gamma^i)} \). Therefore \( d\tilde{k}^i/d\gamma^i > 0 \). From (12) \( dw^i/d\gamma^i > 0 \) therefore \( w^1 > w^2 \).

### 3 Migration

Let us denote by index \( t = 0 \) all variables in autarkic steady-state equilibrium and by index \( t = 1 \) those in steady-state post-migration equilibrium. Assume now that labour migration is permitted at \( t = 0 \), the decision process of an individual is the following. After receiving her inheritance in period \( t = 0 \), a child chooses to migrate (or not) with her wealth. An individual migrates to the country where her indirect utility is the highest. Knowing the autarkic steady-state price system \( p^0 = (w^0_k, R^0_k, \pi^0_k) \), it is possible to study unrestricted migration. Let \( \tilde{U}^{i,k,j}_0 \quad i = 1, 2 \quad k = 1, 2 \quad j = 1, 2 \) be the autarkic steady-state indirect utility of an individual working in country \( i \), born in country \( k \) and of type \( j \). From (5)
\[
\tilde{U}^{i,k,j}_0 = \Omega^{i,k,j}_0 U^j(\alpha_j, 1 - \alpha_j),
\]
where \( \Omega^{i,k,j}_0 = w^0_k + R^0_k A^{i,j}_0 \). Since workers move with their inheritance, \( \Omega^{i,k,j}_0 \) depends on the bequest \( A^{i,j}_0 \) inherited in the country of origin \( k \) and on the price system \( p^0_k \) in the country \( i \) where she works. Thus the difference between the indirect utility of an individual
\( j, \ j = 1, 2 \), born in \( k \) with the bequest \( A_{0}^{k,j} \) if she works in the foreign country \( i \) or in the country of origin \( k \) is

\[
\tilde{U}_{0}^{i,k,j} - \tilde{U}_{0}^{k,k,j} = (\Omega_{0}^{i,k,j} - \Omega_{0}^{k,k,j})U^{ij}(\alpha_{j}, 1 - \alpha_{j}).
\]

As in Galor (1986), if an individual decides to migrate during childhood, she stays in the country of destination during adulthood. Incentives for migration exist when the individual’s indirect utility is higher in country \( i \) than in the country of origin \( k \), that is

\[
\tilde{U}_{0}^{i,k,j} > \tilde{U}_{0}^{k,k,j}.
\]

In autarkic steady-state equilibrium, the price system of economy \( i \), \( p_{0}^{i} = (w_{0}^{i}, r_{0}^{i}, \pi_{0}^{i}) \) determines the capital-labour ratio. If migration restrictions are relaxed, then a migrant of type \( j \) from country \( k \) to country \( i \) supplies her labour to country \( i \) inelastically, at the competitive wage, \( w_{0}^{i} \), and invests her inheritance, \( A_{0}^{k,j} \), at the interest factor, \( R_{0}^{i} \), of country \( i \). The migrant allocates her life-cycle income between consumption, \( C_{1}^{i,j} \), and the optimal bequest left to her child, \( A_{1}^{i,j} \), so as to maximize her utility function \( U^{ij}(C_{1}^{i,j}, A_{1}^{i,j}) \). Equivalently a migrant solves

\[
\max_{C_{1}^{i,j}, A_{1}^{i,j}} U^{ij}(C_{1}^{i,j}, A_{1}^{i,j}) \quad \text{(23)}
\]

\[
\Omega_{0}^{i,k,j} = C_{1}^{i,j} + A_{1}^{i,j} \geq 0, \quad A_{1}^{i,j} \geq 0,
\]

where \( \Omega_{0}^{i,k,j} = w_{0}^{i} + R_{0}^{i} A_{0}^{k,j} \) is the life-cycle income at the steady-state price system of country \( i \). A migrant’s demand for consumption and a migrant’s choice of bequest are therefore

\[
A_{1}^{i,j} = \alpha_{j} \Omega_{0}^{i,k,j},
\]

\[
C_{1}^{i,j} = (1 - \alpha_{j}) \Omega_{0}^{i,k,j}.
\]

In period 1, the optimal bequest left to her child is denoted \( A_{1}^{i,j} = \Omega_{0}^{i,k,j} - C_{1}^{i,j} \). It contributes to the supply of capital per capita in country \( i \) in period 1.

**Definition 1** A post-migration equilibrium in country \( i \) is a price system in period 1, \( p_{1}^{i} = (w_{1}^{i}, R_{1}^{i}, \pi_{1}^{i}) \), such that nobody has an incentive to migrate.

**Remark 1** To any price system in country \( i \) corresponds a unique pair \( (d_{1}^{i}, \gamma_{1}^{i}) \) characterising the economy.

**Proposition 2** In a world economy where two countries have identical constant returns to scale production functions and where populations are heterogeneous with respect to the degree of altruism, if migration is unrestricted, then:

1. Incentives for migration cease when capital-labour ratios are equal across countries as well as population densities.

2. All prices are equal across countries.

3. Numbers of each type of migrants are determined.
The proof of Proposition 2 is provided in two steps. From the definition of a post-migration equilibrium, the first step shows that when incentives for migration cease, capital-labour ratios are equal across countries as well as population densities (see Lemma (2)). The second step shows that migration flows are bilateral and determined, and also that both economies are populated in post-migration equilibrium.

**Proof of Proposition 2: first step**
Substituting (10) and (11) in (21), the indirect utility of a non-migrant $j$ of country $k$ is:

$$U_0^{j,k,j} = (w_0^k + R_0^k A_0^{j,k,j}) U_j(\alpha_j, 1 - \alpha_j)$$

The indirect utility of a migrant $j$ from country $k$ to country $i$ is:

$$U_0^{i,k,j} = (w_0^i + R_0^i A_0^{j,k,j}) U_j(\alpha_j, 1 - \alpha_j)$$

Starting from the autarkic price system equilibrium, individuals $j = 1, 2$ are induced to migrate from country $k = 1, 2$ to country $i = 1, 2$ if:

$$U_0^{i,k,j} > U_0^{j,k,j} \iff \Omega_0^{i,k,j} > \Omega_0^{k,k,j} \iff w_0^i + R_0^i A_0^{j,k,j} > w_0^k + R_0^k A_0^{j,k,j}, \quad j = 1, 2.$$

We consider the post-migration equilibrium price system $p_i^1 = (w_i^1, R_i^1, \pi_i^1)$ such that nobody has an incentive to migrate. At this price system the following equalities hold

$$\forall i, \forall k, \forall j \quad U_1^{i,k,j} = U_1^{k,k,j} \iff \begin{cases} w_1^2 - w_1^1 = (R_1^1 - R_1^2) A_0^{1a} \\ w_1^2 - w_1^1 = (R_1^1 - R_1^2) A_0^{1b} \\ w_1^3 - w_1^1 = (R_1^2 - R_1^1) A_0^{2a} \\ w_1^2 - w_1^3 = (R_1^2 - R_1^1) A_0^{2b}. \end{cases}$$

(26)

**Lemma 2** Capital-labour ratios as well as population densities of each country are equal across countries in post-migration equilibrium.

**Proof of Lemma 2** From (24), equalization of bequest $A_0^{i,a} = A_0^{i,b}, \forall i$ is impossible since individuals $a$ and $b$ have different degrees of altruism, and from Proposition 1, $A_0^{1a} \neq A_0^{2a}, A_0^{1a} \neq A_0^{2b}$. Therefore and from (26) post-migration equilibrium price system $p_1^1 = (w_1^1, R_1^1, \pi_1^1)$ is such that wages are equal across countries and interest factors are equal across countries, i.e. $w_1 = w_1^1 = w_1^2$ and $R_1 = R_1^1 = R_1^2$. From (12) and (13) we obtain

$$\begin{cases} (\tilde{k}_1^1)^{\alpha} (d_1^1)^{\beta-1} = (\tilde{k}_2^2)^{\alpha} (d_2^2)^{\beta-1} \\ (\tilde{k}_2^1)^{\alpha-1} (d_1^1)^{\beta} = (\tilde{k}_2^2)^{\alpha-1} (d_2^2)^{\beta} \end{cases} \implies \begin{cases} \left(\frac{\tilde{k}_1^1}{\tilde{k}_2^2}\right)^{\alpha} = \left(\frac{d_2^2}{d_1^1}\right)^{\beta-1} \\ \left(\frac{\tilde{k}_2^1}{\tilde{k}_2^2}\right)^{\alpha-1} = \left(\frac{d_2^2}{d_1^1}\right)^{\beta}. \end{cases}$$

(27)

Dividing previous equalities we obtain successively

$$\frac{\tilde{k}_1^1}{\tilde{k}_2^2} = \frac{d_1^1}{d_2^2}.$$  

(28)

From (28) capital-labour ratio are equal

$$k_1^1 = k_2^2.$$  

(29)

From (27), (28) and (29) we obtain

$$\begin{cases} \tilde{k}_1^1 = \tilde{k}_2^2 \\ d_1^1 = d_2^2 \\ k_1^1 = k_2^2. \end{cases}$$

(30)
This implies that bilateral migration flows cease when capital-labour ratios of each country are equal, i.e. $k_i^1 = k_i^2$, and when population densities are equal, which achieves the proof of Lemma 2.

**Lemma 3** All prices are equal across countries in post-migration equilibrium.

We have already shown that wages and interest rates are equal across countries in post-migration equilibrium. We now show that prices and returns of land are equal across countries. In post-migration equilibrium (16) and (14) yield

$$
\begin{align*}
\pi_1^1 &= (R_1^1 - 1)q_1^1 \\
\pi_1^2 &= (R_1^2 - 1)q_1^2
\end{align*}
$$

and

$$
\begin{align*}
\pi_1^1 &= (1 - \alpha - \beta)\left(\frac{d}{k_1^1}\right)\alpha(d_1^1)\beta \\
\pi_1^2 &= (1 - \alpha - \beta)\left(\frac{d}{k_2^1}\right)\alpha(d_2^1)\beta.
\end{align*}
$$

From (30) we have $\pi = \pi_1^1 = \pi_1^2$. Since $R_1^1 = R_1^2$, and from (31) we obtain $q_1^1 = q_1^2 = q$. Therefore all prices are equal across countries in post-migration equilibrium $^7$.

**Proof of Proposition 2: second step**

The second step shows that the proportions of each type of migrants are equal across countries. In post-migration equilibrium, wages are equal across countries as well as interest factor, and technologies of production

$$
P(R_1^1, \gamma^1) = P(R_2^2, \gamma^2) = 0 \Rightarrow \gamma^1 = \gamma^2 = \gamma = \frac{N_1^{11} + N_2^{21}}{L_i^{11} + L_i^{22}}.
$$

The two countries have equal proportions of individuals of each type in the post-migration equilibrium. Since $d^1 = d_2 = d$ in post-migration equilibrium, we obtain the number of individuals living in post-migration equilibrium in each country, $i$: $L_i^1 = dX^i$. From $\gamma^i$ we obtain the number of type-1 (type-2) individuals living in country $i$ in post-migration equilibrium $N_1^{11} = \gamma^i L_i^1 \ (N_2^{11} = (1 - \gamma^i)L_i^1)$. By differences, from $N_0^{11} (N_0^{12})$, we obtain the number of type-1 (type-2) individuals having migrated from one country $k$ to the other $i$: $N_1^{11} - N_0^{11} \ (N_2^{11} - N_0^{12})$. \hfill \Box

## 4 Welfare comparison between autarky and post-migration

This section compares the welfare of individuals in steady-state post-migration equilibrium with the situation in autarkic steady-state equilibrium$^8$.

**Proposition 3** If migration restrictions are relaxed between two countries whose proportions of the more altruistic type individuals are respectively equal to $\gamma^1$ and $\gamma^2$ in autarky, with $\gamma^1 > \gamma^2$, the welfare implications are the following:

1. Assuming that the two countries have equal population densities in autarky, there is never a Pareto improvement;

---

$^7$In the following, since prices are equal across countries at the steady-state post-migration equilibrium the indexes of country and time are no longer specified.

$^8$By doing so we do not study the evolution of welfare during the transition before steady-state post-migration equilibrium is reached. We only compare the welfare of individuals at the two steady-state equilibria.
2. Assuming that the two countries have equal interest rates in autarky, there is never a Pareto improvement.

3. Otherwise in some cases, a Pareto improvement may occur.

Proof of Proposition 3: $\gamma^1 > \gamma > \gamma^2 \iff R_0^1 < R < R_0^2$. From (12) and (13):

$$w^{\frac{1}{\alpha}} = \beta \frac{1}{\alpha} k d^{\frac{\alpha - 1}{\alpha}},$$

$$R^{\frac{1}{\alpha}} = \alpha \frac{1}{\alpha} k d^{\frac{\alpha}{\alpha}}.$$  

(33)  

(34)

Dividing (33) by (34) and we obtain $w = (R)^{\frac{\alpha}{\alpha}} \frac{\beta(d)^{\frac{1}{\alpha} - \frac{1}{\alpha - d}}}{(\alpha)^{\frac{1}{\alpha}} - 1}$.

From (17) $\Omega^i = \frac{\beta(\alpha)^{\frac{1}{\alpha}}}{(R)^{\frac{1}{\alpha} - \frac{1}{\alpha}} (1 - \alpha_j R)(d)^{\frac{1}{\alpha} - \frac{1}{\alpha - d}}}$.

(35)

When countries exhibit different interest rates as well as different population densities, before labour mobility is permitted, the welfare implications are quite difficult to sketch on the basis of marginal analysis. International capital mobility linked to labour mobility clears factor prices across countries by equalizing interest rates and by relocating population from the high to the low density of population country.

Let us note $\Delta^i = \Omega^i - \Omega_0^i, \forall i = 1, 2, \forall j = 1, 2$ the welfare differential between the autarkic steady-state equilibrium and the steady-state post-migration equilibrium. Individuals $j$ living initially in country $i$ are better-off in steady-state post-migration equilibrium if following conditions are satisfied

$$\Delta^i > 0 \iff \left( \frac{R_0^i}{R_1^i} \right)^{\frac{1}{\alpha}} (1 - \alpha_j R_0^i) \left( \frac{d_0^i}{d_1^i} \right)^{\frac{1}{\alpha} - \frac{1}{\alpha - d}} > 1.$$  

(36)

The impact of the interest rate effect depends on the individual’s degree of altruism. The density effect applies to all individuals, regardless of their degree of altruism.

These effects are analysed separately in the two following subsections. Subsection 4.2 assumes equal interest rates across countries in autarky. Therefore migration affects only densities in both economies and welfare implications are determined. Subsection 4.1 assumes equal population densities across countries in autarky. Therefore migration affects only the interest rates of both economies. Generally, the welfare conclusions are unclear since several effects are counteracting. We first study two extreme assumptions and show that in some cases the results are unambiguous. Subsection 4.3 presents a situation in which migration leads to a Pareto-improvement.

4.1 Equal densities

We compare the welfare in steady-state post-migration equilibrium with the autarkic steady-state equilibrium for each type of individual assuming equal population densities. Table (1) summarizes the interest factor effect. From (35): $d\Omega > 0 \iff R^i > \frac{\alpha}{\alpha}$. We present all possible cases depending on the parameters values.

Therefore no situation can be a Pareto Improvement after migration.
Table 1: Welfare Results with equal densities

<table>
<thead>
<tr>
<th>Cases</th>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta_{11}$</td>
<td>$\Delta_{12}$</td>
</tr>
<tr>
<td>$1 &lt; \alpha/a &lt; \alpha/b &lt; R^1_0 &lt; R^1 &lt; R^2_0 &lt; 1/a$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$1 &lt; \alpha/a &lt; R^1_0 &lt; \alpha/b &lt; R^1_0 &lt; R^2_0 &lt; 1/a$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$1 &lt; \alpha/a &lt; R^1_0 &lt; R^1_0 &lt; R^2_0 &lt; 1/a$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$1 &lt; R^1_0 &lt; \alpha/a &lt; \alpha/b &lt; R^1 &lt; R^2_0 &lt; 1/a$</td>
<td>$? &lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$1 &lt; R^1_0 &lt; \alpha/a &lt; R^1_0 &lt; \alpha/b &lt; R^2_0 &lt; 1/a$</td>
<td>$? &lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$1 &lt; R^1_0 &lt; \alpha/a &lt; R^1_0 &lt; \alpha/b &lt; R^2_0 &lt; 1/a$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$1 &lt; R^1_0 &lt; R_1 &lt; \alpha/a &lt; \alpha/b &lt; R^2_0 &lt; 1/a$</td>
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<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

4.2 Equal interest factors

This subsection studies the population density effect assuming equal interest rates across countries in autarky. From (36) individuals are better-off in a less crowded country. Two cases are possible:

Case 1: $d^1 < d < d^2$ The individuals living initially in country 1 (2) are worse-off (better-off) after migration, i.e. $\Delta_{11} < 0, \Delta_{12} < 0$, and ($\Delta_{21} > 0, \Delta_{22} > 0$).

Case 2: $d^2 < d < d^1$ The individuals living initially in country 2 (1) are worse-off (better-off) after migration, i.e. $\Delta_{21} < 0, \Delta_{22} < 0$, and ($\Delta_{11} > 0, \Delta_{12} > 0$).

No situation can lead to a Pareto improvement after migration. The less populated country in autarky is more densely populated after migration, which decreases the welfare of its inhabitants.

4.3 Pareto Improvement

Let us consider two countries having different interest rates in autarky such that $1 < \alpha/a < \alpha/b < R^1 < R < R^2 < 1/a$ and different population densities: $d^1 < d^2$. If densities were equal across countries, integration would imply: in country 1, $\Delta_{11} > 0, \Delta_{12} > 0$. In country 2, $\Delta_{21} < 0, \Delta_{22} < 0$. Since we assume $d^1 < d < d^2$, a second effect through density results in a welfare increase of all individuals residing in country 2 and a welfare decrease of all individuals residing in country 1. We can always choose $d^1$ and $d^2$ such that:

1. the positive effect through density more than compensates the negative interest effect for individuals staying in country 2,

2. the negative effect through density less than compensates the positive interest effect for individuals staying in country 1.

Therefore all individuals having or not migrated will be better-off after integration.

5 Discussion of the basic assumptions

Let us now examine more closely the role of various assumptions of the basic model. What happens if we relax the assumption of costless migration (Subsection 5.1) or if economies have non-degenerate wealth distributions (Subsection 5.2)?
5.1 Migration costs

Suppose that migration occurs at a positive fixed cost \( f \). Suppose further that a migrant is constrained to finance the migration on her own wealth. This means that individuals cannot borrow to migrate. This assumption is consistent with the presence of capital market imperfections\(^9\). In this case, an individual \( j \) born in country \( k \) can migrate to country \( i \) only if \( f \leq A^{k,j} \).

5.1.1 Incentive for migration

As in section 3 an incentive for migration exists if \( \tilde{U}^{i,k,j}_0 > \tilde{U}^{k,k,j}_0 \). Suppose without any loss of generality that \( \gamma^1 > \gamma^2 \). From Lemma 1 we have \( w^1 > w^2 \) and \( R^1 < R^2 \). Migration from country 1 to country 2 occurs when

\[
\begin{align*}
\begin{cases}
  w^2 + R^2(A^{1,j} - f) > w^1 + R^1A^{1,j}, \\
  f \leq A^{1,j},
\end{cases} & \iff
\begin{cases}
  w^1 - w^2 < (R^2 - R^1)A^{1,j} - R^2 f, \\
  f \leq A^{1,j}.
\end{cases}
\end{align*}
\]

Migration from country 2 to country 1 occurs when

\[
\begin{align*}
\begin{cases}
  w^1 + R^1(A^{2,j} - f) > w^2 + R^2A^{2,j}, \\
  f \leq A^{2,j},
\end{cases} & \iff
\begin{cases}
  w^1 - w^2 - R^1 f > (R^2 - R^1)A^{2,j}, \\
  f \leq A^{2,j}.
\end{cases}
\end{align*}
\]

System (37) indicates that migration from country 1 to country 2 occurs if the liquidity constraint is satisfied and if the loss in wage is lower than the gain in capital return net of the total cost of migration. Similarly system (38) suggests that migration from country 2 to country 1 occurs if the liquidity constraint is satisfied and if the loss in capital return is more than compensated by the positive wage differential net of migration costs.

5.1.2 Post-migration Equilibrium

Introducing migration costs modifies the qualitative nature of the equilibrium. Indeed each type of individuals, \( j = a, b \) born in \( i = 1, 2 \) has a reservation cost for migration, \( f^{i,j} \), defined by:

\[
\begin{align*}
  f^{1,a} &= \frac{w^2 - w^1 + (R^2 - R^1)A^{1,a}}{R^2} \\
  f^{1,b} &= \frac{w^2 - w^1 + (R^2 - R^1)A^{1,b}}{R^2} \\
  f^{2,a} &= \frac{w^1 - w^2 + (R^1 - R^2)A^{2,a}}{R^1} \\
  f^{2,b} &= \frac{w^1 - w^2 + (R^1 - R^2)A^{2,b}}{R^1}.
\end{align*}
\]

Therefore migration is possible if and only if the following inequalities hold simultaneously:

\[
\begin{cases}
  f \leq f^{i,j}, \\
  f \leq A^{i,j},
\end{cases} \iff f \leq \min \{ f^{i,j}, A^{i,j} \}.
\]

The first inequality relates the existence of a reservation cost and the second one the liquidity constraint.

**Discussion** Introducing migration costs changes migration flows. Indeed, agents of type \( a \) born in country 1 do not migrate to country 2 if the following condition is true:

\[
w_1^1 - w_1^2 \geq (R_1^2 - R_1^1)A^{1,a} - R_1^2 f
\]

\(^9\)Galor and Zeira (1993) make the assumption that a borrowing individual can evade debt payments by migrating.
Similarly, agents of type \( a \) born in country 2 do not migrate to country 1 if the following condition is verified:
\[
 w_1^1 - w_1^2 \leq (R_1^2 - R_1^1)A^{2,a} + R_1^1 f
\]
Consequently, when the last individual of type \( a \) born in country 1 who migrates in country 2 equalizes her indirect utilities across countries, there is no migration from type \( a \) individuals born in country 2 towards country 1. It follows that for type \( a \) individuals, we always have simultaneously an equality and an inequality:
\[
 \begin{cases}
 w_1^1 - w_1^2 = (R_1^2 - R_1^1)A^{1,a} - R_1^1 f \\
 w_1^1 - w_1^2 \leq (R_1^2 - R_1^1)A^{2,a} + R_1^1 f,
\end{cases}
\]
Migration of type \( a \) individuals born in country 1 ceases while individuals of type \( a \) in country 2 do not move (and similarly for type \( b \) individuals).

**Remark** This result still holds if we now consider that migration requires a fixed cost to live in foreign country. Henceforth we drop the assumption on liquidity constraints in home country. With similar reasoning we can easily show that a post-migration equilibrium for type \( a \) individuals verifies necessarily the following equality and inequality:
\[
 \begin{cases}
 w_1^1 - w_1^2 = (R_1^2 - R_1^1)A^{1,a} - f \\
 w_1^1 - w_1^2 \leq (R_1^2 - R_1^1)A^{2,a} + f,
\end{cases}
\]
and similarly for type \( b \) individuals. Consequently, whatever the country, where the positive fixed migration cost is supported, migration flows are changed. There is never a post-migration equilibrium from migration flows maid bilaterally by individuals of the same type. Another implication is that a country never completely vanishes into the other.

### 5.2 Wealth distribution and migration

So far we considered the case of a very simple distribution of wealth in both economies inhabited by two types of individuals. In this subsection, we extend the basic model to include a more general wealth distribution. Let us assume that an economy in autarky is inhabited by altruistic individuals whose degrees of altruism \( j \) follow a distribution \( g \). The distribution of types determines the wealth distribution since bequests depend on individuals’ degrees of altruism. Compared to the basic model, differences come from equations (9) and (11) which are now written as:
\[
 L^i = \int_0^{N^i} g^i(j) dj
\]
\[
 q^i_t X^i_t + K^i_t = \int_0^{N^i} A^i_{t-1}(j)g^i(j) dj
\]
It is clear that equation (40) can be written using (6), (7), (8), (16), (17), (18) which yields equation (41). The implicit function determines the steady-state interest rate in autarky as a function of the distribution of degrees of altruism. \( R^i \) is indeed solution of
\[
 P_D(R^i) = 1 + \frac{\pi(1 - \alpha - \beta)R^i}{\alpha(R^i - 1)} - \frac{\beta}{\alpha} R^i \int_0^{N^i} \frac{j}{1 - j R^i} g^i(j) dj = 0
\]
Let us consider two economies such that initially country 1 has a right-skewed distribution and country 2 a left-skewed distribution. Both economies are identical otherwise. This
captures the fact that country 1 is populated by a higher density of more altruistic type individuals than country 2.

Using (41), a similar reasoning shows that the autarkic steady-state interest rate is higher in economy 2 than in economy 1. The intuition is that the economy which is relatively more dense in very altruistic individuals (with a right-skewed distribution) accumulates more capital than the other. In autarkic steady-state equilibrium interest rate of economy 1 is lower than economy 2 and similarly wage rate is higher in economy 1 than in 2.

We turn now to analyse the incentives for migration. Type $j$ individual in country 2 with initial wealth $A_2^{-j}$ is incited to migrate to 1 if and only if $w^1 - w^2 > (R^2 - R^1)A_2^{-j}$. We can therefore determine a threshold level of initial wealth $\tilde{A}_2 = (w^1 - w^2)/(R^2 - R^1)A_2^{-j}$ below (above) which individuals are (not) incited to migrate. Therefore poor individuals migrate to high capital-labour country while rich individuals prefer to stay in their domestic country.

In economy 2 rich individuals prefer not to migrate since the wage differential does not compensate for the loss due to lower capital returns. But, poor individuals gain to migrate. In economy 1, results are opposite. Therefore poor individuals migrate to economy 1 which is intensive in capital while rich individuals migrate to economy 2. Henceforth the proportions of individuals of each degree of altruism tend to converge across economies until equalization of wage rates and interest factors. Then incentives for further migration cease whatever the individuals initial wealth. The post-migration equilibrium is characterized by equal population densities and equal prices across countries as in the basic version of the model with two types of individuals.

If we add positive migration costs to the present generalization of the model, the two countries remain inhabited in post-migration equilibrium (as in the model with only two types). Migration costs determine a threshold level of degrees of altruism below which individual cannot afford migrating. The results for post-migration equilibrium are the same as in section 5.1. If migration costs are very high, nobody can migrate. Whatever the situation, no country vanishes into the other.

6 Conclusion

In this paper we have studied the pattern of capital mobility linked to labour mobility in an overlapping generations model with land and heterogeneous altruistic individuals. The post-migration equilibrium of the integrated economy is unique and characterized by equal interest rates and equal population densities across countries. This fully determines the numbers of migrants of each type migrating to the other economy.

Two driving forces foster migration. On one hand, individuals have an incentive to migrate from crowded countries to less inhabited ones. On the other hand, individuals have an incentive to migrate which depends on their degree of altruism and the prevailing interest rates in each economy. Capital mobility linked to labour mobility equalizes factor prices across countries and relocates people from high to low density countries.

Comparing the autarkic steady-state welfare with the welfare in the steady-state equilibrium of the integrated world economy leads to the following results. Generally integration of economies has counteracting effects on the welfare of some individuals so that total effects are ambiguous.

Nevertheless in two extreme cases it is possible to generate unambiguous conclusions. If
the two economies exhibit equal densities in autarky, world integration cannot lead to a Pareto improvement in both economies. There is always at least one type of individual in one country who is worse-off after integration. If the two economies have equal interest rates in autarky, world integration has unambiguous effects on the welfare of all individuals in both economies. Individuals residing in the less crowded economy in autarky are worse-off after world integration, whereas individuals in the crowded economy are better-off. This is so because world integration equalizes densities across countries.

Moreover, we show that Pareto-improvement may result. This occurs when the effect through the population density is compensated by the interest rate effect.

References
Population Economics, 5:113-134